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CONVERGING FINITE-STRENGTH SHOCKS

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The converging shock problem was first solved by Guderley and later by Landau and Stanyukovich for infinitely strong shocks in an ideal gas with spherical and cylindrical symmetry. This problem is solved herein for finite-strength shocks and a non-ideal-gas equation of state with an adiabatic bulk modulus of the type $B_S = -v\partial p/\partial v|_S = (p + B)f(v)$, where B is a constant with the dimensions of pressure, and f(v) is an arbitrary function of the specific volume. Self-similar profiles of the particle velocity and thermodynamic variables are studied explicitly for two cases with constant specific heat at constant volume; the Tait-Kirkwood-Murnaghan equation, f(v) = constant, and the Walsh equation, f(v) = v/A, where A = constant. The first case reduces to the ideal gas when B = 0. In both cases the flow behind the shock front exhibits an unbalanced buoyant force instability at a critical Mach number which depends upon equation-of-state parameters.

INTRODUCTION

The convergence of a strong shock in a perfect gas is a well-known problem in hydrodynamics. The shock profile is self-similar when the shock front propagates at infinite Mach number into a uniform perfect gas at rest in spherical or cylindrical geometry. When the density ahead of the shock is a power-law function of the radius, the shock convergence problem is also self-similar in the limit of infinite Mach number; see Chernous'ko (1960).

Here we solve the shock convergence problem for an arbitrary Mach number and for more general equations of state than that of the ideal gas. The solutions give the rate of approach to the asymptotic solution as well as the dependence of the shock motion and profile upon the equation of state. A remarkable result is that the flow behind the shock goes unstable at a critical Mach number even for perfect spherical symmetry. This critical Mach number depends on the equation of state of the medium. The instability itself is driven by unbalanced buoyant forces.

The key ideas for the solution are Lie-group invariance and parametric separation of variables. The scale transformations which determine self-similar motions are Lie-group operations. Birkhoff (1950) first applied group theory to find invariant solutions of Euler's equations. Subsequently, several others, for example, Ovsjannikov (1962), Michal (1951), and Müller and Matschat (1962) have refined Lie's original method of integration of differential equations by group theoretical techniques.

Adiabatic fluid motion is governed by Euler's equations which contain three independent dimensions, mass, length, and time. They can admit three independent scale transformations. Self-similar shock motions utilize all three independent choices of scale. One choice is fixed by the initial density ahead of the shock. Another is determined by the numerical solution for the similarity exponent. This also determines the shock trajectory in space-time and the shape of the flow behind the shock. The remaining choice of scale can be used to determine by a scale transformation the profile at a later time from its profile at an earlier time. Self-similar motions in one dimension satisfy ordinary differential equations in scale-invariant variables rather than partial differential equations in space and time. They are important physically because they represent asymptotic states of motion which occur when the fluid is no longer strongly influenced by its initial conditions.

Self-similar motions are not possible for arbitrary equations of state. Generally, the bulk modulus, $B_S(p,v)$, in terms of which Euler's equations can be written, removes some scale freedom by connecting the pressure p with the specific volume v. Initial and boundary conditions also tend to remove scale freedom by introducing characteristic dimensions.

Lie-group invariance can be used to determine forms of the equation of state for which self-similar solutions of Euler's equations will exist. The equations of state determined from the Lie-group invariance condition include a class of Mie-Grüneisen equations for which the temperature and volume dependence of the free energy is separable additively. Thus, the theory applies to solids, for example, at shock pressures well above the yield stress, but at compressions where the volume dependence of the specific heat is negligible.

Self-similar solutions for converging shocks in media with more general equations of state than that of the ideal gas have been found by Axford and Holm (1978) for the case of shocks with infinite strength. In the present study quasi-similar solutions for converging spherical and cylindrical shocks with finite strength in media described by the Walsh form of the adiabatic bulk modulus are obtained for a range of material parameters and shock-front Mach numbers.

The occurrence of the Mach number in the shock front boundary conditions would seem to preclude a self-similar reduction for an arbitrary Mach number. However, an approximate separation of the similarity variable from the Mach number does occur because the shock boundary data is specified along a noncharacteristic constant value of the similarity variable with Mach number as a parameter. This separation leads to a self-similar reduction at each value of the Mach number that was discussed for the ideal gas first by Oshima (1960) and later by Lee (1967). The solution from this parametric self-similar reduction is a generalization of the Guderley solution to include Mach-number dependence. As in the Guderley solution, there is a critical sonic line which determines the similarity eigenvalue at each Mach number. Because this critical-point structure of the solution persists for an arbitrary Mach number, the limit to the Guderley solution is uniform.

In the following we first state the problem and describe the method of approach. Results from an invariance analysis of Euler's equations are summarized next. These results are then used to determine equations of state for which self-similar solutions exist. Two such equations of state are those of Tait-Kirkwood-Murnaghan (see MacDonald (1969)) and of Walsh (1961), which have been used previously as empirical interpolation functions without the realization that self-similar solutions exist for them. After these equations of state are identified, the quasi-similar reduction of Euler's equations is discussed, and the numerical method used to solve the quasi-similar problem is described. In the last section numerical results are given.

STATEMENT OF PROBLEM AND THE GEOMETRIC APPROACH TO ITS SOLUTION

Consider the problem of the spherical or cylindrical convergence of a shock wave to the center of a uniform stationary material. The initial conditions ahead of the shock are

$$p = p_0, \quad \rho = \rho_0, \quad u = 0. \tag{1}$$

At the shock front the boundary conditions are given by the Rankine-Hugoniot jump conditions, which, with D the shock front velocity and E the specific internal energy, are

$$u/D = 1 - v/v_0, \tag{2}$$

$$p - p_o = \rho_o uD, \tag{3}$$

and

$$E - E_o = (p + p_o)(v_o - v)/2 = u^2/2 + p_o(v_o - v). \tag{4}$$

The terms in p_o are ignored in the case of a strong shock.

In the absence of viscosity and heat transfer, the motion of the shock front and the flow behind the shock are governed by Euler's equations. These are the continuity equation,

$$d\rho/dt + \rho(u_r + su/r) = 0, \tag{5}$$

the equation of motion,

$$du/dt + p_r/\rho = 0, \tag{6}$$

and the energy or entropy equation,

$$dp/dt + B_s(p,\rho)(u_r + su/r) = 0, \tag{7}$$

where r is the spatial coordinate, d../dt is the substantial time derivative, and $s = 0,1,2$ is the usual geometrical factor.

The pressure-volume response of the material is described by the adiabatic bulk modulus defined by

$$B_s(p,v) = -v\partial p/\partial v|_S. \tag{8}$$

The equation of state enters the formulation through the adiabatic bulk modulus in (8). When written in terms of a general adiabatic bulk modulus, an invariance analysis leads to the construction of self-similar solutions and other types of invariant solutions for shocks in media other than an ideal gas. We seek functional forms of the adiabatic bulk modulus for which Euler's equations admit the maximal group of point transformations.

INVARIANCE PRINCIPLES FOR EULER'S EQUATIONS

Euler's equations (5) to (7) admit a three-parameter subgroup of scale transformations generated by the operator,

$$Q_{op} = (2a_1 + a_3)r\partial_r + (a_1 + a_3)t\partial_t + a_1u\partial_u + (a_2 - 2a_1)\rho\partial_\rho + a_2(p + B)\partial_p, \tag{9}$$

provided that the adiabatic bulk modulus satisfies the condition,

$$a_2(p + B)\partial B_s/\partial p + (a_2 - 2a_1)\rho\partial B_s/\partial\rho - a_2B_s = 0, \tag{10}$$

where B is a constant with units of pressure. The general solution of (10) is

$$B_s(p,\rho) = (p + B)f[(p + B)^{1 - 2a_1/a_2}/\rho], \tag{11}$$

where f is an arbitrary function of the indicated argument. When the adiabatic bulk modulus has this form, three independent scale transformations are admitted by Euler's equations, which are also invariant under time translations, since the independent variable t does not appear explicitly in the system. The time origin can be chosen arbitrarily. In planar geometry invariance under spatial displacements and Galilean transformations would also occur. In more spatial dimensions rigid rotations of all vectors would be admitted.

A relation must be imposed in Q_{op}, namely,

$$a_2 - 2a_1 = 0, \tag{12}$$

in order for the initial condition ahead of the shock to be invariant. In this case the adiabatic bulk modulus for self-similar shock propagation into a uniform medium assumes the separable form,

$$B_s(p,\rho) = (p + B)f(\rho), \tag{13}$$

in which $f(\rho)$ is an arbitrary function of the density. Such equations for the adiabatic bulk modulus have been used previously as interpolation functions in shock-wave physics. Two choices for $f(\rho)$ are well-known; for the TKM equation, we have

$$f(\rho) = \text{constant} = 1/(A\rho_o) = \Gamma, \tag{14}$$

and for the Walsh equation we have

$$f(\rho) = \text{const.}/\rho = 1/A\rho = \Gamma\rho_o/\rho. \tag{15}$$

The Walsh equation for the adiabatic bulk modulus has the additional advantage that it is consistent with the experimentally observed linear relation between the shock speed D and the particle speed u behind the shock, namely,

$$D = c + s_H u, \tag{16}$$

which is true for plate-impact experiments in which the shock pressure is greater than approximately fifty kilobars. In terms of the parameters in (16), the constants A and B in the Walsh form for the adiabatic bulk modulus are given by

$$A = v_o/4s_H = 1/4s_H\rho_o, \quad B = \rho_o c^2/4s_H. \tag{17}$$

For metals the number s_H is typically about 1.25, and the number c is roughly equal to the speed of sound.

CONSTRUCTION OF SIMILARITY VARIABLES AS GROUP INVARIANTS

Euler's equations can be reduced to a system of three nonlinear ordinary differential equations by a transformation of the variables to the invariant coordinates of the operator Q_{op}, which are solutions of the linear partial differential equation,

$$Q_{op} f(r,t,u,\rho,p) = 0. \tag{18}$$

In general, the solution of such a first order partial differential equation involves arbitrary functions of the functionally independent integrals of the set of characteristic equations. In our case these arbitrary functions can be taken as new dependent variables in Euler's equations whose solutions are restricted to invariant surfaces. The flow variables that are determined from the independent group invariants are as follows;

$$\lambda = r/t^\alpha, \; u = (r/t)U_s(\lambda), \; \rho = \rho_o R_s(\lambda), \; p + B = (r/t)^2\rho_o P_s(\lambda), \tag{19}$$

where the exponent α is given by

$$\alpha = (2a_1 + a_3)/(a_1 + a_3), \tag{20}$$

and the value of the time is taken to be negative before the collapse and to vanish when the shock reaches the center.

Upon the substitution of the self-similar flow variables into Euler's equations, a coupled set of three nonlinear ordinary differential equations in λ remains to be integrated. The initial and boundary conditions for this system will be invariant if the shock trajectory follows the path represented by

$$r_H(t) = \text{const. } t^\alpha, \tag{21}$$

and also if the initial density distribution is uniform, namely,

$$\rho(x,0) = \rho_o. \tag{22}$$

Details of the self-similar shock convergence problem for shocks with infinite strength in media described by a Walsh adiabatic bulb modulus have been reported by Axford and Holm (1978). In the next section we shall discuss an approximate solution for converging shocks with finite strength that limits to the infinite strength case as the Mach number of the shock front approaches infinity.

QUASI-SIMILAR FLOWS FOR FINITE-STRENGTH SHOCKS

For a finite-strength shock the shock-front Mach number and the similarity exponent are interconnected and depend upon time. The finite-strength jump conditions depend upon the Mach number of the shock front relative to the speed of sound in the material ahead of the shock. This Mach number changes with time along the curved shock trajectory $r_H(t)$. Thus, in a self-similar solution the similarity exponent depends upon the shock-front Mach number through the jump conditions, which are boundary conditions for the flow behind the shock.

An approximate solution, in which a finite-strength shock would follow a series of instantaneously self-similar states, can be sought. In this solution the Mach number of the shock front enters parametrically, and the problem can be solved by a series of self-similar steps. The approximate solution is of self-similar form, but it is rescaled with the shock-front Mach number to match the finite jump conditions at the shock front. The compression, for example, is of the form,

$$\rho/\rho_o = R_1(\lambda)R_2(M), \quad \lambda = r/t^{\alpha(M)}, \tag{23}$$

where the Mach-number-dependent scale factor, $R_2(M)$, is the ratio of the Rankine-Hugoniot conditions for finite and infinite Mach numbers which limits to unity from below as the Mach number approaches infinity. In the case of an ideal gas we have

$$R_2(M) = (\gamma - 1)/(\gamma - 1 + 2/M^2), \tag{24}$$

where γ is the specific heat ratio. When trial solutions of the factorized form for the dependent variables are substituted into Euler's equations, these equations reduce to ordinary differential equations in λ with the Mach number as a parameter. The factorized approximate solution affords a self-similar reduction for each value of the shock-front Mach number. A similarity exponent can be obtained that corresponds to the Mach number of the shock front located at a given position. This similarity exponent can then be used to determine the Mach number and position of the shock front at a slightly later time. This procedure can be iterated to follow the evolution of a converging flow through a series of instantaneously self-similar states. Such a solution can be regarded only as an approximate solution because it is strictly valid only near the shock front, since the Mach-number dependence originates in the jump conditions. However, a general solution would be expected to evolve toward a self-similar solution asymptotically. Approximate solutions of the type under consideration were first proposed by Oshima (1960) to include counterpressure in the analysis of a blast wave and were called by him quasi-similar solutions. Later, Rae (1970) and Lee (1967) investigated aspects of quasi-similar solutions for converging shocks and detonations in an ideal gas.

Quasi-similar solutions for the two non-ideal-gas equations of state considered herein are calculated from the change of variables that follows. Euler's equations for these equations of state assume the form,

$$d\rho/dt + \rho(u_r + su/r) = 0, \tag{25}$$

$$du/dt + p_r/\rho = 0, \tag{26}$$

and

$$\rho dp/dt + \Gamma(p + B)(\rho_o/\rho)^{w-1}(u_r + su/r) = 0, \tag{27}$$

where $s = 1$ or 2 for cylindrical or spherical geometry, respectively, and $w = 1$ or 2 for the TKM or the Walsh equation of state, respectively. Let us define new independent variables by

$$z = r/r_H(t), \quad M = \dot{r}_H/c_o, \tag{28}$$

where $r_H(t)$ is the shock-front locus to be determined, and the shock-front Mach number M is taken relative to the speed of sound c_o in the medium ahead of the shock. The self-amplification coefficient defined by

$$\theta(t) = \ddot{r}_H r_H/\dot{r}_H^2 = d\ell n\, M/d\ell n\, r_H \tag{29}$$

is a function of the time, or Mach number of the shock front, alone. To transform dependent variables, set

$$\rho = \rho_o R(z,M), \tag{30}$$

$$u = c_o MzX(z,M), \tag{31}$$

and

$$p + B = \rho_o c_o^2 M^2 z^2 Y(z,M), \tag{32}$$

and define the vector $\underline{v}$ as

$$\underline{v} = \begin{pmatrix} R \\ X \\ Y \end{pmatrix}. \tag{33}$$

Then, with $x = \ell n(z)$ and $y = \ell n(M^{1/\theta})$, Euler's equations take the normal characteristic form with the components defined as

$$v^i_y + A^i_j(\underline{v})v^j_x = K^i(\underline{v}), \tag{34}$$

where the matrix $A^i_j(\underline{v})$ and the right-hand side of (34) are given explicitly by

$$A^i_j(\underline{v}) = \begin{bmatrix} X-1 & R & 0 \\ 0 & X-1 & 1/R \\ 0 & \Gamma Y/R^{w-1} & X-1 \end{bmatrix} \tag{35}$$

and

$$-K^i(\underline{v}) = \begin{pmatrix} (s+1)XR \\ \theta X + X(X-1) + 2Y/R \\ 2\theta Y + 2(X-1) + (s+1)\Gamma YX/R^{w-1} \end{pmatrix}, \tag{36}$$

and we sum over repeated indices.

The characteristic directions of the transformed Euler equations are found from

$$\det[A^i_j(\underline{v}) - dx/dy\,\delta^i_j] = 0. \tag{37}$$

Since Euler's equations are hyperbolic, there are three characteristic directions, which are defined by

$$dx/dy = X - 1, \quad dx/dy = X - 1 \pm \sqrt{\Gamma Y/R^w}. \tag{38}$$

These characteristics correspond to particle trajectories and incoming and outgoing sonic lines. They also correspond to roots of

$$\Delta_o = \det A^i_j(\underline{v}) = 0, \tag{39}$$

which determine the critical points of the quasi-similar solutions at a given Mach number of the shock front.

To obtain the quasi-similar type of solution a factorization ansatz is made for each component of the vector $\underline{v}$; that is, we write

$$v^i(y,x) = v^i(y,0)v^i(-\infty,x)/v^i(-\infty,0), \tag{40}$$

where the Rankine-Hugoniot conditions for a finite Mach number of the shock front are given by

$$\lim_{x\to 0} v^i(y,x) = v^i(y,0), \tag{41}$$

and the truly self-similar solution is contained in the limit,

$$\lim_{y\to -\infty} v^i(y,x) = v^i(-\infty,x), \tag{42}$$

for an infinite Mach number . Under this factorization ansatz the transformed Euler equations reduce to ordinary differential equations, namely,

$$A^i_j(\underline{v})dv^j/dx = K^i - D^i_j(y)v^j, \tag{43}$$

where $D^i_j(y)$ is a diagonal matrix that takes the values which follow. For the Walsh form of the adiabatic bulk modulus, with the definitions,

$$\varepsilon = (1+\delta)/M^2, \quad \delta = p_o/(p_o + B), \tag{44}$$

we have

$$D_{11}(\varepsilon) = -4\varepsilon/(\Gamma - 2 + 2\varepsilon), \tag{45}$$

$$D_{22}(\varepsilon) = -2\varepsilon/(1-\varepsilon), \tag{46}$$

and

$$D_{33}(\varepsilon) = -\varepsilon/(1-\varepsilon/2), \tag{47}$$

together with the Walsh boundary conditions,

$$R(z=1,M) = (-2/\Gamma + 2\varepsilon/\Gamma)^{-1}, \tag{48}$$

$$X(z=1,M) = 2(1 - \varepsilon)/\Gamma, \tag{49}$$

and

$$Y(z=1,M) = 2(1 - \varepsilon/2)/\Gamma. \tag{50}$$

For the TKM form of the adiabatic bulk modulus, with the definition,

$$\varepsilon' = 1/M^2, \tag{51}$$

the diagonal matrix elements are given by

$$D_{11}(\varepsilon') = -4\varepsilon'/(\Gamma - 1 + 2\varepsilon'), \tag{52}$$

$$D_{22}(\varepsilon') = -2\varepsilon'/(1 - \varepsilon'), \tag{53}$$

and

$$D_{33}(\varepsilon') = -\varepsilon'/(\Gamma/(\Gamma - 1) - \varepsilon'/2), \tag{54}$$

and the corresponding TKM boundary conditions are given by

$$R(z=1,M) = (\Gamma + 1)/(\Gamma - 1 + 2\varepsilon'), \tag{55}$$

$$X(z=1,M) = 2(1 - \varepsilon')/(\Gamma + 1), \tag{56}$$

and

$$Y(z=1,M) = 2(1 - (\Gamma - 1)\varepsilon'/2\Gamma)/(\Gamma + 1). \tag{57}$$

After this quasi-similar reduction to ordinary differential equations, the remaining part of the solution follows the method used for predicting the motions of self-similar collapsing shocks. That is, each of the derivatives

$$dv^i/dx = \Delta_i/\Delta_o \tag{58}$$

where Δ_o is defined in (39), is found with Cramer's rule, which defines the three additional determinants denoted by Δ_i. The differential equations (58) are then integrated numerically with a fourth or fifth order Runge-Kutta algorithm.

NUMERICAL METHOD OF SOLUTION

As in the Guderley solution only two of the four determinants Δ_0, Δ_1, Δ_2, Δ_3 are linearly independent. Therefore, the critical-point nature of the true self-similar solution is preserved by the quasi-similar transformation, and the similarity exponent for each value of the shock-front Mach number is determined from the existence of a critical sonic line that follows the shock front. Accordingly, the similarity exponents are computed as functions of both material parameters, for example, Γ as defined in (15), and the Mach number of the shock front.

The numerical criterion for the determination of correct values of the similarity exponent originates in the linear dependence of any two of the four determinants Δ_0, Δ_1, Δ_2, Δ_3 upon the other two. For specified values of the material parameters and the shock-front Mach number the corresponding value of the similarity exponent is found by a computer interactive shooting method in which the locus of the plot of Δ_0 versus Δ_1 is required to pass within a specified distance of the origin. To ensure that this locus passes within 10^{-6} units of the origin requires that the value of the similarity exponent sought be determined to a precision of seven figures

NUMERICAL RESULTS

Numerical values of the similarity exponent for fourteen values of the shock-front Mach number are given in Table 1 for spherically collapsing shocks and in Table 2 for cylindrically collapsing shocks in media described by the Walsh form of the adiabatic bulk modulus. These numerical values were computed with the trial-and-error procedure discussed in the previous section. In the limit of a very large value of the shock-front Mach number, say 10000, the values of the similarity exponents shown in Tables 1 and 2 have been found to agree with those values computed with a separate code which was written for the infinite shock-strength case in which the shock-front Mach number does not appear as a parameter as it does in the finite shock-strength case.

The results that are presented in Tables 1 and 2 provide the data required to take into explicit account the effect of finite-shock strengths in relating two positions of the shock front during the process of convergence. From the definition of the self-amplification coefficient given in (29), it follows directly that

$$dr_H/r_H = dM/\theta M. \tag{59}$$

Consequently, the integral in

$$r_H/r_{Hi} = \exp \int_{M_i}^{M} (M'\theta)^{-1} dM', \tag{60}$$

where r_{Hi} is the initial position of a shock front with the Mach number M_i, can be evaluated numerically to give the shock-front position corresponding to the Mach number M.

Pressure, density, and velocity profiles can be obtained easily with the similarity exponents given in Tables 1 and 2. Because of space limitations in the present paper we shall present elsewhere numerical results for the similarity exponents for media with the TKM form of the adiabatic bulk modulus, explicit shock trajectories and profiles, and a discussion of the stability properties of flows in media with a Walsh and with a TKM form of the adiabatic bulk modulus.

Table 1. Variation of the similarity exponent with the shock-front Mach number for spherically collapsing shocks in media with a Walsh adiabatic bulk modulus.

M ↓	SIMILARITY EXPONENT $\alpha(M,\Gamma)$		
	Γ = 8.0	Γ = 5.0	Γ = 3.0
10000.00	.6174999	.6512684	.7548466
100.00	.6175324	.6512931	.7548406
10.00	.6207583	.6537491	.7544191
4.47	.6338886	.6639045	.7536002
3.16	.6505163	.6771068	.7544990
2.24	.6844786	.7051091	.7619652
1.83	.7193817	.7350854	.7759424
1.58	.7552483	.7668846	.7954111
1.41	.7921338	.8004175	.8195537
1.29	.8301372	.8356759	.8477553
1.20	.8694230	.8727576	.8796416
1.12	.9102782	.9119233	.9151370
1.05	.9532780	.9537683	.9546700
1.03	.9759736	.9761160	.9763739

Table 2. Variation of the similarity exponent with the shock-front Mach number for cylindrically collapsing shocks in media with a Walsh adiabatic bulk modulus.

M ↓	SIMILARITY EXPONENT $\alpha(M,\Gamma)$		
	Γ = 8.0	Γ = 5.0	Γ = 3.0
10000.00	.7578689	.7858619	.8584830
100.00	.7578937	.7858800	.8584804
10.00	.7603517	.7876845	.8582484
4.47	.7702659	.7950779	.8578996
3.16	.7826190	.8045347	.8586926
2.24	.8071884	.8240716	.8638262
1.83	.8315728	.8442980	.8728756
1.58	.8557785	.8650806	.8850401
1.41	.8798233	.8863266	.8996954
1.29	.9037351	.9079865	.9163796
1.20	.9275793	.9300581	.9347848
1.12	.9514327	.9526016	.9547617
1.05	.9754612	.9757820	.9763599
1.03	.9876331	.9877184	.9878719

In the evaluation of (60) with these two tables, the relation

$$\theta = 1 - 1/\alpha \tag{61}$$

between the self-amplification coefficient and similarity exponent is used.

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